Aliquot sequences

Active research site

back to the front page

go directly to the page of open problems

go directly to the page of database

go directly to the fundamental database on aliquot sequences

go directly to the page to see our attempt to disprove Catalan's conjecture

see directly our work on the evolution of the average growth coefficient of the aliquot sequences based on the number of iterations

Welcome to all those who are interested in our active research site dedicated to aliquot sequences.

A subject of that kind has been dealt with by lots of other sites (click on the link to see a few examples) but our site presents only unpublished works that cannot be found on the other above-mentioned sites.

The whole text has been drafted in French but an English version of certain pages should be progressively published, which is understandable due to the translation cost of great quantities of text.

This page translated into English is meant to summarize hereunder the main advances of our team.

The site was launched on September 8th 2010, and hence could highlight the enormous stride forward in getting more familiar with the aliquot sequences.

What was presented here as the second conjecture of Garambois has become Barbulescu-Garambois' theorem i.e.:

There is an increasing aliquot sequence at each iteration of a factor at least k during i successive iterations, with k and i having any magnitude whatsoever.

The demonstration by Razvan Barbulescu of the theorem can be found by clicking on the link. Click here to see the article of Erdös which is quoted at the end of Razvan Barbulescu's demonstration.

And what was presented here as the third conjecture of Garambois has become Chtaibi-Garambois' theorem i.e.:

A guide in an aliquot sequence is all the more likely to be preserved with iterations going along as the terms of the aliquot sequence are getting bigger.

The demonstration by Youssef Chtaibi of the theorem can be found by clicking on the link.

In actual fact, the main purpose of the site lies in the last part entitled "open problems", because we need help to boost research further since certain questions appear to be difficult to us and others require a good deal of patience in order to draft programmes and mainly run them so as to expect the results !
The second main purpose of this site is our database, for those who need it. Some data are often updated.

Conjectures will also be presented, some of which might be turned into demonstrated theorems while others might be reformulated or given up as research will go along.

We won't define here again an aliquot sequence because it has been well done on other English sites.

The main advances presented on the site are not mentioned anywhere else :

1) The notion of isolated aliquot cycle put forward by Jean-Luc Garambois

An aliquot cycle may have one link (perfect numbers like 6, 28 or 496), two links (amicable pairs like 220 and 284) or even more.
An isolated aliquot cycle is an aliquot cycle on which no aliquot sequence starting on a number that does not belong to this chain, can come out.
Thus, 6 is a perfect number which is not isolated on the infinite graph of the aliquot sequences because the aliquot sequence that is starting at the integer 25, comes out at the perfect number 6.
On the other hand, 28 is an isolated perfect number on the infinite graph of the aliquot sequences because it only has one aliquot antecedent i.e. itself and no other.
Any aliquot cycle whose all link-constituting integers have strictly just one aliquot antecedent, are therefore isolated aliquot cycles. Several amicable pairs can be identified today as two-link isolated aliquot cycles. (Ask for list).
Except for those amicable pairs and the perfect number 28, no other isolated aliquot cycle has been known so far, but there must be a lot of them to be discovered, some of them having 2 links and more.

2) The notion of aliquot sequence with a high coefficient of growth put forward by Jean-Luc Garambois

The best would be to explain it by giving examples.

The integer 19560 increases whenever a k factor at least minimal to 2, is iterated. Paul Zimmermann has carried on calculations up to iteration 565 and numbers of 197 figures can be reached without any alteration.

As to the integer 620542913760, the k factor which is higher than 3 remains lower than 3 up to iteration 234 where a number of 129 figures will be reached.

The integer 9900243648828670255121203200 = 215×35×52×72×11×13×17×19×31×43×211×257×303997 retains a k growth factor higher or equal to 4 on 97 successive iterations.

Quite a lot of other examples are presented in this part of the site, up to k=8...

Let's also highlight how interesting it is to study the sequences having coefficients higher or equal to 1 i.e. strictly growing.

For k=1 and k=2, we have sequences that keep those minimal growth factors as far as our calculations could be made.
For k higher or equal to 3, the factor always remains lower or equal to 3 after a certain number of iterations thanks to the calculating capacities of modern computers.
Nevertheless, sequences for which k will remain higher or equal to 3 should be found, as far as our calculations might be made in the near future.

Do note that the k minimal growth factor that has be chosen for a sequence needn't be necessarily taken as an integer, but the latter cases have only been studied by our team so far.

A highly effective method meant to research aliquot sequences with high growth coefficients has been empirically found by Jean-Luc Garambois:

An aliquot sequence that is growing at each iteration by a factor at least equal to the k integer on i iterations will be carried out. To do so, the smallest integer m which is (k+1)-perfect will be used and the aliquot sequences having the n=zm with z taking up all the whole values extending as far as necessary to obtain the desired k on the i number of desired iterations, will be tested.

3) An aliquot sequence tends to preserve a "guide" on several successive iterations i.e. a phenomenon observed by quite a lot of people, but well-argued by our team

Preserving guide 2 which is a driver too
If an even number n is taken as the start of an aliquot sequence, the successive terms of the aliquot sequence "will tend" to remain even, for only the perfect squares and their doubles will allow the parity to be changed when the function "sum of the aliquot parts" (sigma' or σ') is applied. See the demonstration. Driver 2 of the aliquot sequence is also said to be preserved.
Perfect squares or their doubles being just considered, implies an increased scarcity of the numbers whereby a change of parity can be obtained through the σ' function when greater and greater integers are to be considered.

Preserving guide 3
If all integers n that can be divided by 3, those whose σ'(n) cannot be divided by 3 are all and without any exception numbers denoted as

"Loeschian numbers" i.e. x^2+xy+y^2, with x and y as integers. This has been conjectured by Jean-Luc Garambois and demonstrated by Razvan Barbulescu (see the demonstration). But be careful : the reverse is not necessarily true: there are Loeschian numbers that can be divided by 3 and which do not lose their divisibility by 3 when function σ' is applied. Moreover, Razvan Barbulescu has demonstrated that the integers n that can be divided by 3 and which can't remain the same when σ' function is applied, would also become scarcer when ever-increasing integers are to be considered : see the demonstration.

Preserving guide p, with p as a prime number higher than 3

The increased scarcity of the n numbers that can be divided by p and with p as a prime number higher than 3 and which do not preserve the guide p when the σ' function is applied, can only be observed thanks to a computer and cannot be demonstrated at all when ever-increasing numbers are to be considered. Click here to see the numerical arguments.

Preserving composed guides

The scarcity of the n numbers that do not preserve composed guides such as 8=2^3 can also be observed if ever-increasing integers are to be considered. Click here to see numerical arguments.

This has finally encouraged us to formulate Garambois'N°3 conjecture :

A guide in an aliquot sequence is all the more likely to be preserved with iterations going along as the terms of the aliquot sequence are getting bigger.

This conjecture has been proven by Youssef Chtaibi : click here.
Razvan Barbulescu verified this demonstration.

Exceptional preservation of certain particular guides

Drivers 6=2*3 or 120=2^3*3*5 or the guide 30240=2^5*3^3*5*7 seem to preserve themselves significantly better than the others of the same order of size as iterations go along in an aliquot sequence. Click here to see numerical arguments. The guides are not second-rate since they are respectively the smallest 2-perfect, 3-perfect and 4-perfect numbers.

This has finally encouraged us to formulate Garambois'N°4 conjecture :

In the terms of an aliquot sequence, a k-perfect guide can preserve itself significantly better than any other guide of the same order of size.

It would be essential to succeed in characterising all n numbers multiple of one of these particular guides such that σ'(n) also remains a multiple of the same guides although this seems to be somehow difficult. However it should be noted that it is the case for all n integers of the type n=qp, with q as a perfect number and p as a prime number different from those decomposing the q guide. See the demonstration. But these are not the

only numbers which explain why over half these guides' multiples remain very quickly multiples of the guides when the function σ' is applied.
*Let's remind once more that the ability of self preservation in an aliquot sequence is not that much stressed for the non k-perfect guides of the same order of size, except seemingly for some of them that are the products of quite tiny prime numbers such as 360=2^3*3^2*5 and which have quite a high number of divisors.*

The aim to reach would be to be able to calculate how probable it would be to preserve any given composed guide in an aliquot sequence on i iterations, depending on the size of the n integer which is starting the aliquot sequence, on i and also on the decomposition of the guide into prime factors. The guide involves a certain minimal k growth factor of the aliquot sequence on those i iterations.

It is actually to reach that point that we have been concentrating all our efforts towards searching polynomial shapes generating the n numbers multiple of p prime whose divisibility by p cannot be preserved when the σ' function is applied. One starts from the notion that it will be easier to find the "scarcity laws" of these numbers with polynomial shapes than without them although this may be sure.

4) Shapes of the n numbers that do not preserve the prime guides higher than 3 or the composed guides.

Only the x^2 et $2x^2$ shaped numbers with x integer do not preserve the 2 factor (or the guide or the driver) when the σ' function is applied.

All the numbers that do not preserve guide 3 when the σ' function is applied, will be x^2+xy+y^2 shaped with x and y as integers.

Which are the shapes of the numbers that do not preserve the other prime numbers (5, 7...) ?

Which are the shapes of the numbers that do not preserve the composed guides and more specifically those we are mostly interested in i.e. the particular guides that are better preserved than the ones mentioned here-above ?

Such a research appears to be difficult and is at the root of several open problems that are presented on our site.

5) Other novel works presented on the site

Numerous other open problems linked to the aliquot sequences are also presented on the site, including challenges for programmers, and are to be solved with the help of computers.

The site also presents works where one has not just tried to apply the function σ' to integers, but also to relative integers, Gauss complex integers and even polynomials… (See the works).
Our attempt towards generalizing has been made so as to find rules that are more general and which could generate new properties on aliquot sequences, invisible when one remains restricted within the frame of natural integers.

Other iteration processes such as $n \rightarrow \sigma'(n)-1$ (divisor 1 is no longer taken into consideration for numbers while iterating) or more generally $n \rightarrow \sigma'(n)+b$ with b as a relative integer, have also been briefly studied within that framework.

All these generalizing attempts have not brought about convincing results for the time being, but the studies may prove quite interesting "per se".

Please also pay attention at the end of this English summary of the site, that "successful" methods and programmes are put forward to determine for instance the number of aliquot antecedents of the integers (Please click here to download some data on our data base) or to search for isolated aliquot cycles or to see our fundamental database on aliquot sequences.

All the results that have been given and brought forward are well-argued and the line of reasoning that has generated them is presented therein.

Good luck to all those who wish to take part in such an adventure !